SUZANNE CORY HIGH SCHOOL

Using the TI-Nspire Notes page for open-ended assessments

James Lam

Felicity Hung

QR code for resources:

- CAS Notes page examples
- Year 11 Methods problem-solving task example

QR code for Padlet:

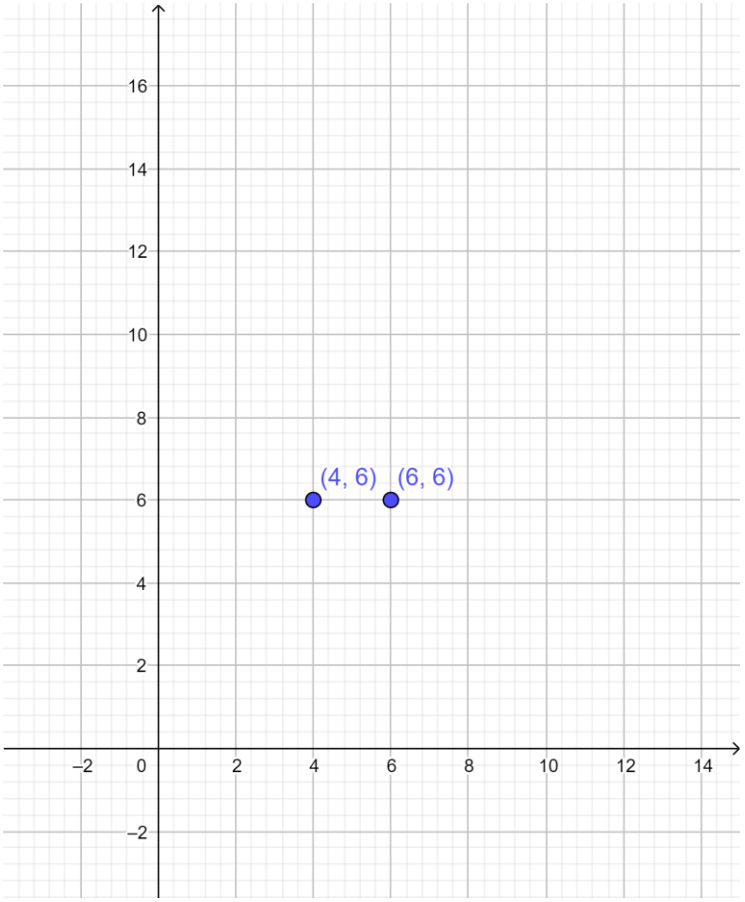
- Share your ideas with us and your peers here!

Part 1.1 Marking Scheme

Question 2 (15 marks)

Texas places two traffic cones at $(4, 6)$ and $(6, 6)$. Cassie starts at the point $(0, 0)$ and has been instructed to construct a parabola which:

- Can navigate around and above the traffic cones (and not hit them)
- Has another x-intercept where its x-coordinate is between 8 and 12.
- Has a turning point with a y-coordinate less than 15.



a) Create a quadratic function such that it fulfills the criteria above.

Check function against conditions (use CAS file if necessary)

- $f(4) > 6 \; and \; f(6) > 6$ A1
- x-intercept other than $(0, 0)$ is between $x = 8$ and $x = 12$. (Inclusive; accept $x = 8$ and $x = 12$) A1
- Turning point of y-coordinate < 15 A1

Your quadratic function is:

$f(x) =$

Part 1.1 Notes page example

Input quadratic function:

$$f(x):=\frac{-x^2}{2}+5\cdot x \quad \text{Done}$$

Working space:

Condition Checks

Above cones at (4, 6) & (6, 6)

$f(4)>6 \text{ and } f(6)>6 \quad \text{true}$

x−intercept between x=8 and x=12

$8\le \text{right}(\text{solve}(f(x)=0,x)|7<x<13)\le 12 \quad \text{true}$

Turning point below y=15

$f(\text{right}(\text{fMax}(f(x),x)))<15 \quad \text{true}$

Part 1.2 Marking Scheme

b) Using your created quadratic function f, evaluate:

i) $f(4)$ Correct value A1

ii) $f(6)$ Correct value A1

c) Sketch your quadratic function on the graph in part a). Label all axial intercepts and the turning point with their coordinates.

- Correct inverted parabolic shape A1
- Both x-intercepts labelled A1
- Turning point labelled A1

Texas is about to draw a line with the equation $y = kx + 40$. Before he draws it, he asks Cassie for a value of k such that this line will be a **tangent** to the graph of f.

d) Find the value of k correct to one decimal place.

Evidence of finding the discriminant of correct equation. M1
i.e. Discriminant of $0 = f(x) - kx - 40$

Let $\Delta = 0$ for 1 solution. Solve for values of k.

Correct negative value of k. A1

Part 1.2 CAS Notes page example

Outputs

bi. $f(4) \to 12$

bii. $f(6) \to 12$

c. $TP=(xtp:=\text{right}(\text{fMax}(f(x),x)) \to 5, f(xtp) \to \frac{25}{2})$

d. expanded equation $\text{expand}(f(x)-k\cdot x-40) \to \frac{-x^2}{2}-k\cdot x+5\cdot x-40$ k=

$k1:=\text{right}(\text{solve}(b^2-4\cdot a\cdot c=0,k)|k<0) \to -3.9442719$

Other

$a:=\text{list}\blacktriangleright\text{mat}(\text{polyCoeffs}(f(x)-k\cdot x-40))[1\quad 1] \to \frac{-1}{2}$

$b:=\text{list}\blacktriangleright\text{mat}(\text{polyCoeffs}(f(x)-k\cdot x-40))[1\quad 2]$
$\to -(k-5)$

$c:=\text{list}\blacktriangleright\text{mat}(\text{polyCoeffs}(f(x)-k\cdot x-40))[1\quad 3] \to -40$

Part 1.3 Marking Scheme

The function f has been dilated from the x-axis by a factor of $\frac{3}{2}$ to create the function g.

e) For the function g, find the new locations of the:

i) Starting point

$(0, 0)$ A1

ii) Turning point

Correct coordinates A1

f) Does the function g fulfill all three criteria specified for function f?
Explain your answer for each criteria.

Criteria	**Explanation**
Can navigate around and above the traffic cones (and not hit them)	Students should answer YES. Explanation: Show $g(4) > 6$ and $g(6) > 6$ explicitly, or describe this qualitatively. **(A1 for both YES and explanation)**
Has another x-intercept where its x-coordinate is between 8 and 12.	Students should answer YES. Explanation: State correct x-intercept that's between $x = 8$ and $x = 12$. **(A1 for both YES and explanation)** Should be the same as function f.
Has a turning point with a y-coordinate less than 15.	Students can answer YES or NO, depending on their function g. If NO, then $y \geq 15$. If YES, then $y < 15$. **(A1 for NO/YES and explanation)**
Does g fulfill all 3 criteria? (Circle one)	YES / NO (No mark for this)

Part 1.3 CAS Notes page example

eii. $\left[\text{right}\left(\text{fMax}\left(f(x),x\right)\right) \to 5,\ \frac{3}{2}\cdot f(\text{xtp}) \to \frac{75}{4}\right]$

fi. (above cones?)

$\frac{3}{2}\cdot f(4)>6 \text{ and } \frac{3}{2}\cdot f(6)>6 \to \text{true}$

fii. (x-int between 8 and 12?)

$\text{right}\left(\text{solve}\left(\frac{3}{2}\cdot f(x)=0,x\right)\middle|x>0\right) \to 10$

fiii. (Is the TP lower than y=15?)

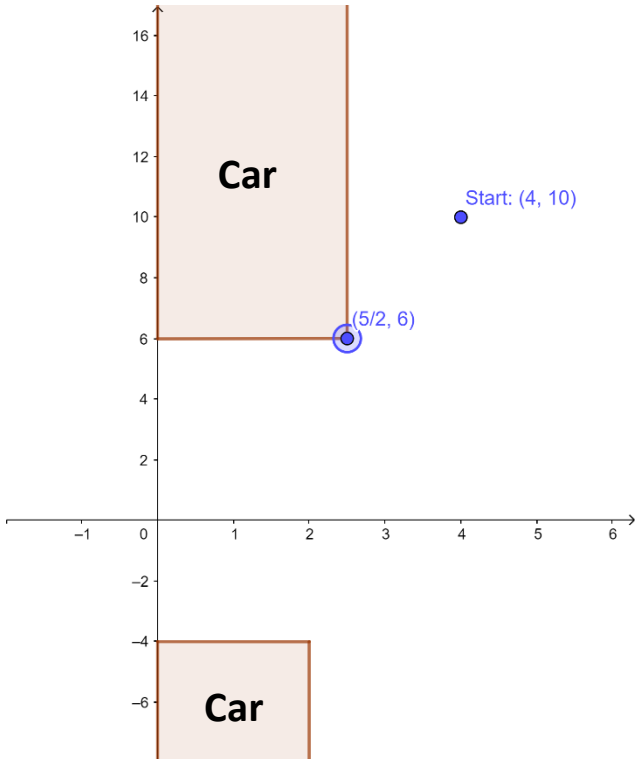
$\frac{3}{2}\cdot f\left(\text{right}\left(\text{fMax}\left(f(x),x\right)\right)\right) \to 18.75$

Sample task and marking scheme (Functions Investigation Part 2, Question 2)

Try creating a CAS file which can assist with marking student responses to the task below.

Question 2 (11 marks)
Check CAS file for answers.

Cassie wants to try to reverse parallel park as well, starting from the point $(4, 10)$. However, the parked car beside her starting point is larger than the case in **Question 1**; she will need to ensure she avoids hitting this car.



Texas has instructed Cassie to create a cubic function f_1 in stationary point of inflection form:

$$f_1(x) = a(x-h)^3 + k$$

The following criteria must be satisfied for the new function f_1:

- Its right endpoint is $(4, 10)$.
- Its left endpoint is its x-intercept, which has an x-coordinate between $\frac{1}{2}$ and 2.
- The curve must be below the point $\left(\frac{5}{2}, 6\right)$ when $x = \frac{5}{2}$.
  i.e. The path must not hit the parked car.

The width of Cassie's car may be considered as negligible.
i.e. Width of car = 0 m.

a) Find a rule for the new cubic function, f_1.

3 marks in total; check for conditions satisfied:

- The rule passes through $(4, 10)$ A1
- The left x-intercept is between $\frac{1}{2}$ and 2 inclusive. A1
- $f\left(\frac{5}{2}\right) < 6$ A1

*Note: Maximum of 1 mark if the function created is not a cubic.

*Note: Q2b – 2g are consequential to the student's function in Q2a.

b) State the coordinates of the x-intercept of the graph of f_1, correct to 2 decimal places.

Correct x-intercept in coordinate form. A1
Must be in coordinate form, not written as $x = a$.

c) State the domain of f_1 correct to 2 decimal places, assuming both endpoints are inclusive.

Correct domain. A1

d) State the coordinates of the stationary point of inflection of f_1.

Correct stationary point of inflection. A1

e) Sketch the graph of $y = f_1(x)$ on the image in part a).
Label endpoints and stationary points of inflection with their coordinates correct to 2 decimal places.

3 marks in total:

- Shape A1
- Stationary point of inflection A1
- Endpoints labelled A1/2 x2
  Don't penalise open/closed endpoints.

The function f_1 is translated k units to the left to form a new function, f_2.

f) If f_2 passes through $\left(\frac{5}{2}, 6\right)$, find the value of k correct to 2 decimal places.

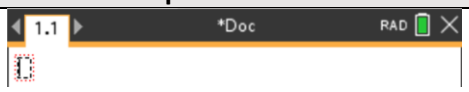
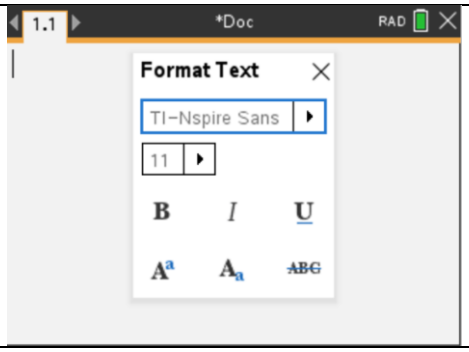
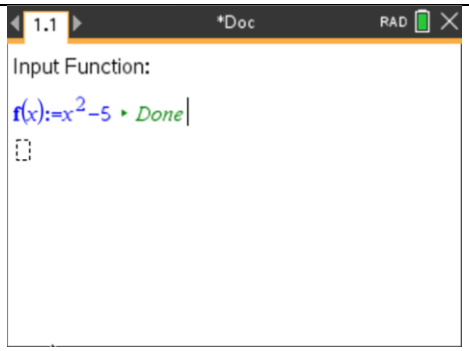
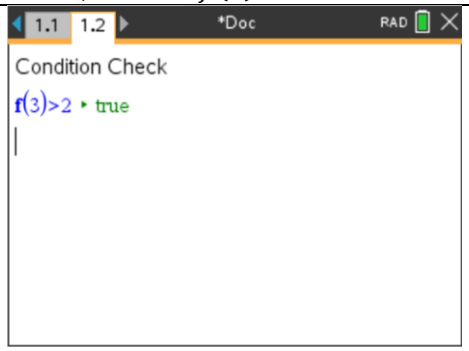
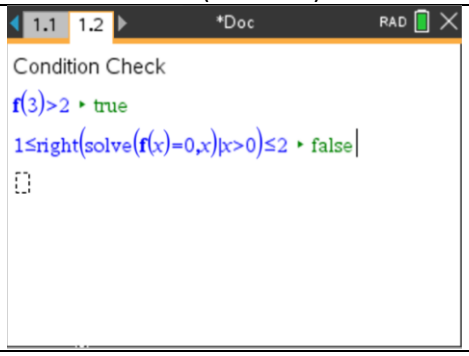
Correct value of k A1

g) Hence or otherwise, find all **positive** values of k where the graph of f_2 allows Cassie to **not** hit the large parked car.

Correct interval for k A1

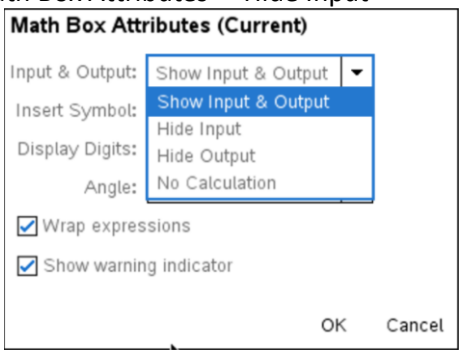
Glossary

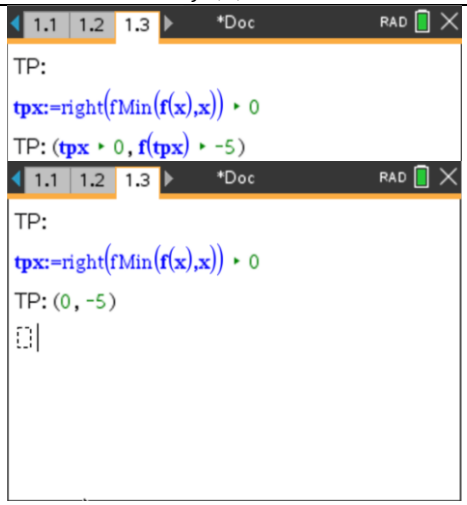
Note: All the instructions below require "Note" page using the TI-Nspire.

TI-Nspire CAS Keystrokes	TI-Nspire CAS Screens
To insert a Maths box, press ctrl M. All commands from the Calculator page can be accessed in a Maths box.	
To adjust the font size and font style, press menu > **4 Format** > **1 Format text…**	
Define a function: Define $f(x) = x^2 + 5$	
Inside a Maths box, enter the function name with :=. Press enter when the action is done. Obtain := by pressing ctrl : Ensure the instruction is clear for others to follow, a quick instruction such as "input function" can be labelled before defining a function. In this case, you do not need to use Maths box.	
Check whether a simple condition is true: For $f(x) = x^2 - 5$, check if $f(3) > 2$.	
Inside a Maths box, enter the condition that needs to be checked. If the condition is true, a ▸ true is shown. If the condition is false, a ▸ false is shown.	
Check the right-/left-hand side of the equation is true: For $f(x) = x^2 - 5$, check if the positive x-intercept is between 1 and 2 (inclusive).	
In this case where x-intercept(s) needs to be solved, an output $x = \pm\sqrt{5}$ is obtained when solving $f(x) = 0$. To check whether one of the solutions ($\sqrt{5}$ in this example) is true, only the right-hand side of the equation needs to be checked. Therefore, type $right()$ into a Maths box with the condition that is required to be checked.	

Show/Hide input or output: To show the turning point of the function $f(x) = x^2 - 5$.

To view only the turning point's coordinates (i.e., (0,-5)), press (menu) > 5 Math Box Options > 1 Math Box Attributes > Hide Input





List a Coefficient: Find the coefficient of x for $g(x) = (3x - 2)^2$.

Use the catalogue key (book) to access polyCoeffs(for listing the coefficients of a polynomial.

Use the catalogue key (book) to access list▸mat(which converts a list into a row matrix.
$[a \quad b]$ traces the element in the a^{th} row and the b^{th} column of this matrix.

In this case, the coefficient of x is the second coefficient of the polynomial, the value in the first row and the second column is listed.